Tables of Transforms

Table I. Fourier Cosine Transforms

	$f(x)$	$\hat{f}_c(w) = \mathcal{F}_c(f)$
1	$\begin{cases} 1 & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$	$\sqrt{\frac{2}{\pi}}\,\frac{\sin aw}{w}$
2	$x^{a-1} \quad (0 < a < 1)$	$\sqrt{\frac{2}{\pi}}\,\frac{\Gamma(a)}{w^a}\cos\frac{a\pi}{2}$ $\quad$ ($\Gamma(a)$ see App. A3.1.)
3	$e^{-ax} \quad (a > 0)$	$\sqrt{\frac{2}{\pi}}\left(\frac{a}{a^2 + w^2}\right)$
4	$e^{-x^2/2}$	$e^{-w^2/2}$
5	$e^{-ax^2} \quad (a > 0)$	$\frac{1}{\sqrt{2a}}\,e^{-w^2/(4a)}$
6	$x^n e^{-ax} \quad (a > 0)$	$\sqrt{\frac{2}{\pi}}\,\frac{n!}{(a^2 + w^2)^{n+1}}\,\mathrm{Re}\,(a + iw)^{n+1}$ $\quad$ Re = Real part
7	$\begin{cases} \cos x & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$	$\frac{1}{\sqrt{2\pi}}\left[\frac{\sin a(1 - w)}{1 - w} + \frac{\sin a(1 + w)}{1 + w}\right]$
8	$\cos(ax^2) \quad (a > 0)$	$\frac{1}{\sqrt{2a}}\cos\left(\frac{w^2}{4a} - \frac{\pi}{4}\right)$
9	$\sin(ax^2) \quad (a > 0)$	$\frac{1}{\sqrt{2a}}\cos\left(\frac{w^2}{4a} + \frac{\pi}{4}\right)$
10	$\frac{\sin ax}{x} \quad (a > 0)$	$\sqrt{\frac{\pi}{2}}\,(1 - u(w - a))$ $\quad$ (See Sec. 6.3.)
11	$\frac{e^{-x}\sin x}{x}$	$\frac{1}{\sqrt{2\pi}}\arctan\frac{2}{w^2}$
12	$J_0(ax) \quad (a > 0)$	$\sqrt{\frac{2}{\pi}}\,\frac{1}{\sqrt{a^2 - w^2}}\,(1 - u(w - a))$ $\quad$ (See Secs. 5.5, 6.3.)

Table II. Fourier Sine Transforms

	$f(x)$	$\hat{f}_s(w) = \mathcal{F}_s(f)$
1	$\begin{cases} 1 & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$	$\sqrt{\frac{2}{\pi}}\left[\frac{1 - \cos aw}{w}\right]$
2	$1/\sqrt{x}$	$1/\sqrt{w}$
3	$1/x^{3/2}$	$2\sqrt{w}$
4	$x^{a-1} \quad (0 < a < 1)$	$\sqrt{\frac{2}{\pi}}\,\frac{\Gamma(a)}{w^a}\sin\frac{a\pi}{2}$ ($\Gamma(a)$ see App. A3.1.)
5	$e^{-ax} \quad (a > 0)$	$\sqrt{\frac{2}{\pi}}\left(\frac{w}{a^2 + w^2}\right)$
6	$\frac{e^{-ax}}{x} \quad (a > 0)$	$\sqrt{\frac{2}{\pi}}\arctan\frac{w}{a}$
7	$x^n e^{-ax} \quad (a > 0)$	$\sqrt{\frac{2}{\pi}}\,\frac{n!}{(a^2 + w^2)^{n+1}}\,\mathrm{Im}(a + iw)^{n+1}$ Im = Imaginary part
8	$xe^{-x^2/2}$	$we^{-w^2/2}$
9	$xe^{-ax^2} \quad (a > 0)$	$\frac{w}{(2a)^{3/2}}e^{-w^2/4a}$
10	$\begin{cases} \sin x & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$	$\frac{1}{\sqrt{2\pi}}\left[\frac{\sin a(1 - w)}{1 - w} - \frac{\sin a(1 + w)}{1 + w}\right]$
11	$\frac{\cos ax}{x} \quad (a > 0)$	$\sqrt{\frac{\pi}{2}}\,u(w - a)$ (See Sec. 6.3.)
12	$\arctan\frac{2a}{x} \quad (a > 0)$	$\sqrt{2\pi}\,\frac{\sin aw}{w}e^{-aw}$

Table III. Fourier Transforms

See (6) in Sec. 11.9.

	$f(x)$	$\hat{f}(w) = \mathcal{F}(f)$
1	$\begin{cases} 1 & \text{if } -b < x < b \\ 0 & \text{otherwise} \end{cases}$	$\sqrt{\frac{2}{\pi}}\,\frac{\sin bw}{w}$
2	$\begin{cases} 1 & \text{if } b < x < c \\ 0 & \text{otherwise} \end{cases}$	$\frac{e^{-ibw} - e^{-icw}}{iw\sqrt{2\pi}}$
3	$\frac{1}{x^2 + a^2} \quad (a > 0)$	$\sqrt{\frac{\pi}{2}}\,\frac{e^{-a\lvert w\rvert}}{a}$
4	$\begin{cases} x & \text{if } 0 < x < b \\ 2x - b & \text{if } b < x < 2b \\ 0 & \text{otherwise} \end{cases}$	$\frac{-1 + 2e^{ibw} - e^{-2ibw}}{\sqrt{2\pi}w^2}$
5	$\begin{cases} e^{-ax} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \quad (a > 0)$	$\frac{1}{\sqrt{2\pi}(a + iw)}$
6	$\begin{cases} e^{ax} & \text{if } b < x < c \\ 0 & \text{otherwise} \end{cases}$	$\frac{e^{(a-iw)c} - e^{(a-iw)b}}{\sqrt{2\pi}(a - iw)}$
7	$\begin{cases} e^{iax} & \text{if } -b < x < b \\ 0 & \text{otherwise} \end{cases}$	$\sqrt{\frac{2}{\pi}}\,\frac{\sin b(w - a)}{w - a}$
8	$\begin{cases} e^{iax} & \text{if } b < x < c \\ 0 & \text{otherwise} \end{cases}$	$\frac{i}{\sqrt{2\pi}}\,\frac{e^{ib(a-w)} - e^{ic(a-w)}}{a - w}$
9	$e^{-ax^2} \quad (a > 0)$	$\frac{1}{\sqrt{2a}}\,e^{-w^2/4a}$
10	$\frac{\sin ax}{x} \quad (a > 0)$	$\sqrt{\frac{\pi}{2}}$ if $\lvert w\rvert < a$; 0 if $\lvert w\rvert > a$